

NONLINEAR BENDING OF POROUS-CORE SANDWICH FRAMES WITH GPL REINFORCEMENT FACE LAYERS

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Received: 18 March 2026 / Revised: 22 April 2026 / Accepted: 15 May 2026

Published online: 9 June 2026

Abstract. This article presents a numerical investigation into the nonlinear bending behavior of sandwich frames composed of a functionally graded (FG) porous core and graphene platelet-reinforced composite (GPLRC) face layers. Using Timoshenko beam theory and von Kármán geometric nonlinearity, a mathematical model for frames that accounts for various GPL distribution patterns is derived. To ensure numerical efficiency and eliminate the shear-locking problem, coupled high-order shape functions are employed to derive a beam element. Using this element, the discretized nonlinear equilibrium equation is established and solved by an iterative procedure. The accuracy of the proposed frame model is rigorously validated against existing data in the literature, showing excellent agreement across specific cases. Various frames are analyzed to highlight the significant impacts of several key parameters, such as the porosity coefficient and the weight fraction of GPLs, on the nonlinear responses. The influence of the GPL distribution patterns on the nonlinear behavior of the sandwich frames is also explored and discussed.

Keywords: nonlinear bending, sandwich frames, porous material, GPL reinforcement composite, shear-locking, Timoshenko beam.

1. INTRODUCTION

High-performance, lightweight structures for advanced engineering sectors such as aerospace, marine, and automotive industries are in high demand, and this has driven substantial innovations in materials science. To meet this need, researchers have drawn inspiration from nature to develop porous structures that mimic biological forms such as natural bones or timber. By introducing internal pores, structural density becomes a controlled design parameter, providing a way for lightweight forms with strengthened mechanical properties. Functionally graded (FG) porous composites, in which the internal porosity gradation is strategically controlled, have emerged as a promising candidate to enhance structural performance (D. Chen et al., 2023). Extensive efforts have been devoted to understanding the static and dynamic behaviors of these porous materials. For instance, the elastic buckling and static bending analyses of shear deformable FG porous beams have been carried out using the Ritz method by D. Chen et al. (2015), while refined mixed finite element models have been developed by Zghal et al. (2022) to

capture the effects of even and uneven porosity distributions on the bending deflections and stresses.

Despite their advantages, the deliberate introduction of internal pores also leads to a decrease in the overall stiffness and load-bearing capacity of the structures. To effectively overcome this drawback, the use of high-strength nanoscale fillers to reinforce the porous matrix has become a crucial and widely adopted method. Graphene platelets (GPLs) and carbon nanotubes (CNTs) with extraordinary mechanical properties and exceptional specific surface area have consistently proven to be excellent reinforcement materials (Shen & Xiang, 2013; Sitli et al., 2024). Functionally graded graphene platelet-reinforced composite (FG-GPLRC) structures, a new class of high-performance structures, have drawn considerable attention from engineers and researchers in recent years. Studies have demonstrated that incorporating a small weight fraction of GPLs into a matrix, whether utilizing uniform or layer-wise distributions, can remarkably improve structural stiffness and reduce bending deflections (Songsuwan et al., 2023; Tam et al., 2020). The nonlinear bending behavior of FG-GPLRC structures has also been explored by various researchers through analytical and numerical approaches, including the Gram-Schmidt-Ritz method combined with direct iteration techniques for beams (Songsuwan et al., 2023), and finite element analysis for cracked beams (Tam et al., 2020), highlighting the impacts of GPL reinforcement and the sensitivity of the nonlinear responses to external factors such as crack depth and location.

The integration of FG porous materials and GPL reinforcement into the fabrication of structural elements to form FG-GPLRC porous composite structures is a state-of-the-art solution that maximizes the stiffness-to-weight ratio of structures. The nonlinear bending, buckling, and vibration behaviors of such integrated structures have attracted intense academic interest across both macro and micro scales. At the macroscopic level, the geometrically nonlinear static responses and stability of FG porous arches with GPL reinforcement have been investigated analytically by Liu et al. (2019), using the virtual work method. Similarly, Anirudh et al. (2020) utilized a shear-flexible finite element approach to explore the nonlinear flexural bending behavior of porous curved composite beams reinforced by GPLs, while higher-order shear deformation theories accounting for through-thickness stretching have been applied by Polit et al. (2019) to study the bending responses of nanocomposite curved beams. The thermal postbuckling, nonlinear vibration, and nonlinear bending of porous beams were considered by X. Chen et al. (2023) using inhomogeneous models and two-step perturbation methods. At the microscale level, the size-dependent nonlinear behaviors have been rigorously investigated utilizing the nonlocal strain gradient theories (Dang & Do, 2021; Sahmani et al., 2018), the modified couple stress theory (Arshid et al., 2021; Mirjavadi et al., 2017), and the quasi-3D theory combined with the modified strain gradient theories (Karamanli & Vo, 2021) to predict the bending, thermal buckling, and vibration characteristics of FG porous microbeams and microplates.

To achieve optimization of structural efficiency, sandwich construction, typically comprising a lightweight porous core flanked by stiff composite face layers to form high-strength porous sandwich structures, is often applied by engineers and architects. Because these advanced lightweight structures are inherently slender, they are highly susceptible to large deformations under operational loading, making geometrically nonlinear analysis necessary. The mathematical modeling of geometric nonlinearity, particularly through the nonlinear von Kármán strain-displacement relationship, has been widely employed in the analysis of FG beam-like structures (Lin et al., 2018; Niknam et al., 2014; Yang et al., 2018; Zhao et al., 2022). The nonlinear bending of sandwich structures specifically consisting of metal foam cores and FG-GPLRC face-sheets has recently been considered. For example, analytical solutions for such sandwich beams have been derived using the numerically stable Chebyshev-Ritz method by Wang et al. (2022), while generalized beam theories and various micromechanical models have been applied

by Wattanasakulpong (2025) to approximate the effective material properties and to evaluate the responses of the structure under diverse loading conditions. Furthermore, the finite element approach was used with the Newton-Raphson iterative procedure by Nguyen et al. (2018) to predict the elastoplastic nonlinear responses and plastic propagation in FG composite beams loaded by different types of distributed loads.

Despite these extensive studies, a critical gap remains that requires further attention. The vast majority of existing numerical and analytical studies on FG porous structures with GPL reinforcement have been strictly limited to isolated and simple structural elements such as single straight beams, single curved beams, or isolated plates (Anirudh et al., 2020; Wang et al., 2022; Wattanasakulpong, 2025). Practical engineering applications, however, heavily rely on frame structures. The nonlinear bending behavior of structural frames is highly complex due to intricate joint interactions and coupled axial-flexural deformations that isolated beam models cannot accurately predict. Analyzing such interconnected systems using traditional analytical approaches, such as the Ritz or differential quadrature methods, often becomes mathematically difficult when dealing with complex boundary conditions and multi-member configurations. Therefore, developing reliable, efficient finite element (FE) formulations specifically tailored for composite sandwich frames is not only an academic pursuit but also an urgent necessity for the safe and optimized design of next-generation lightweight infrastructure and aerospace structures. Addressing this need, Pham and Le (2025) recently formulated a third-order shear deformable beam element for the nonlinear bending analysis of FG porous beams and frames.

Furthermore, in simulations of slender shear deformable beams and frames, conventional Timoshenko finite element formulations face the "shear-locking" problem. This issue stems from the use of inappropriate shape functions in the derivation of the finite element beam formulation. A shear-locking beam element causes numerical defects, and the modeled structure exhibits an artificially exaggerated stiffness, leading to highly inaccurate deflection predictions and unacceptably slow convergence rates. Failing to address shear-locking in the geometrically nonlinear analysis of highly sensitive structures such as FG-GPLRC porous sandwich frames can lead to serious design miscalculations.

To bridge these critical gaps, this study presents a comprehensive numerical investigation of the nonlinear bending behavior of sandwich composite frames. The frames considered herein consist of an FG porous core and GPL-reinforced composite face layers with different patterns of GPL distribution. The nonlinear mathematical model for the sandwich frames is rigorously constructed, in which both Timoshenko beam theory and von Kármán geometric nonlinearity are employed to capture large deformation kinematics. An advanced finite element formulation, in which the coupled quadratic and cubic shape functions derived by solving linear equilibrium equations are employed to interpolate the sectional rotation and transverse displacement, is derived and used to establish the nonlinear equilibrium equation. These sophisticated interpolations are specifically chosen to eliminate the shear-locking phenomenon, which is commonly encountered in classical Timoshenko beam elements, and to improve the accuracy and convergence of the nonlinear numerical solutions. To verify the reliability, stability, and precision of the proposed finite element formulation, the results obtained by the derived formulation are rigorously compared against available data available in the literature for specific cases. A comprehensive parametric analysis of various frame configurations is performed to evaluate the significant and coupled effects of various key parameters, such as the porosity coefficient and the weight fraction of GPLs, on the nonlinear bending capacity of the sandwich structure. The influence of the patterns of GPL distribution and sandwich configuration on the nonlinear behavior of the sandwich frames is also studied and highlighted.

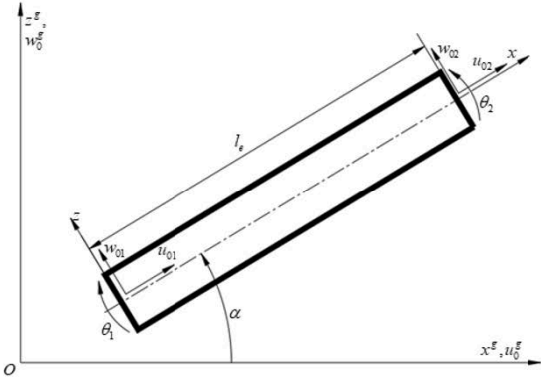
2. MATHEMATICAL FORMULATION

2.1. Sandwich frame with FGP core and FG-GPLRC faces

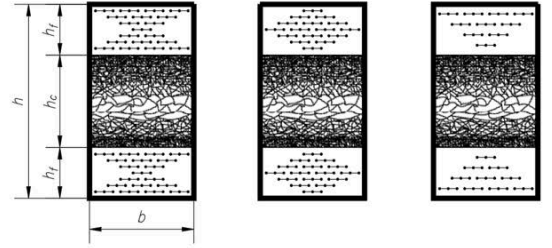
Fig. 1(a) illustrates a generic two-node beam element and its nodal degrees of freedom in local and global coordinate systems, the (x, z) and (x_g, z_g) , respectively. The considered sandwich frame consists of a symmetrically distributed porous core with thickness h_c and two FG-GPLRC face sheets with thickness h_f , with three distribution patterns of GPLs, namely FG-O, FG-X, and FG-V, as described by Wattanasakulpong (2025) (Fig. 1(b)). Accordingly, the total thickness is given by $h = h_c + 2h_f$. The effective material properties of the sandwich beam with different distributions of the constituent phases are first determined in this section. When GPLs are employed as reinforcement in matrix face sheets, the GPL weight fraction, g_{GPL} , serves as an input parameter. From g_{GPL} , the corresponding GPL volume fraction, V_{GPL} , can be calculated as follows (Wattanasakulpong, 2025)

$$V_{GPL} = \frac{g_{GPL}}{g_{GPL} + \left[\frac{\rho_{GPL}}{\rho_M} (1 - g_{GPL}) \right]}. \quad (1)$$

Here, ρ_{GPL} and ρ_M represent the mass densities of the GPLs and the matrix phase, respectively. The corresponding volume fraction of the matrix phase is given by $V_M = 1 - V_{GPL}$. The total volume fraction is continuously graded along the thickness direction, as mentioned above, according to different distribution patterns, the functionally graded X-type (FG-X), O-type (FG-O), and V-type (FG-V), as summarized in Table 1. The associated grading functions, denoted by F_x , F_o , and F_v , are also presented in Table 1.



(a) Beam element



(b) Cross-section with three distribution patterns X, O, and V

Fig. 1. Beam element model for frame structures with an FGP core and two FG-GPL-reinforced face sheets

Once the GPL distributions in the two face sheets are specified, they provide the basis for evaluating the effective material properties of the FG-GPLRC layers. These properties include the thickness-dependent Young's modulus $E(z)$, the mass density $\rho(z)$, and Poisson's ratio $\nu(z)$, which are determined as follows

$$\begin{cases} E^f(z) = \frac{3}{8}E_A^f + \frac{5}{8}E_T^f, \\ \rho^f(z) = V_{GPL}^f \rho_{GPL} + (1 - V_{GPL}^f) \rho_M, \\ \nu^f(z) = V_{GPL}^f \nu_{GPL} + (1 - V_{GPL}^f) \nu_M. \end{cases} \quad (2)$$

Table 1. Equivalent parameters and GPL volume fractions in the face sheets of the sandwich frame

Type	Top layer	Bottom layer	Equivalent condition
FG-X	$V_{GPL}^T = F_x \cdot V_{GPL} \cdot \left(\frac{2z - (h_c + h_f)}{h_f} \right)^2$	$V_{GPL}^B = F_x \cdot V_{GPL} \cdot \left(\frac{2z + (h_c + h_f)}{h_f} \right)^2$	$V_{GPL} - F_x \cdot V_{GPL} \cdot \frac{2}{h - (h_c + h_f)} \cdot \int_{(h_c+h_f)/2}^{h/2} \left(\frac{2z - (h_c + h_f)}{h_f} \right)^2 dz = 0$
FG-O	$V_{GPL}^T = F_o \cdot V_{GPL} \cdot \left[1 - \left(\frac{2z - (h_c + h_f)}{h_f} \right)^2 \right]$	$V_{GPL}^B = F_o \cdot V_{GPL} \cdot \left[1 - \left(\frac{2z + (h_c + h_f)}{h_f} \right)^2 \right]$	$V_{GPL} - F_o \cdot V_{GPL} \cdot \frac{2}{h - (h_c + h_f)} \cdot \int_{(h_c+h_f)/2}^{h/2} \left[1 - \left(\frac{2z - (h_c + h_f)}{h_f} \right)^2 \right] dz = 0$
FG-V	$V_{GPL}^T = F_o \cdot V_{GPL} \cdot \left(\frac{2z - h_c}{h - h_c} \right)$	$V_{GPL}^B = F_o \cdot V_{GPL} \cdot \left(\frac{2z + h_c}{-h + h_c} \right)$	$V_{GPL} - F_o \cdot V_{GPL} \cdot \frac{2}{h - (h_c + h_f)} \cdot \int_{(h_c+h_f)/2}^{h/2} \left(\frac{2z - h_c}{h - h_c} \right) dz = 0$

The superscripts B and T in Table 1 represent the bottom and top face sheets, respectively. The parameters associated with the GPL reinforcement, including the material coefficients η_A and η_T , as well as the geometric factors ξ_A and ξ_T , can be estimated by various micromechanical models, including the Halpin-Tsai, Lusi, and Padawer-Becher models (Wattanasakulpong, 2025). The Halpin-Tsai model is the most widely used for composites with specific nanoparticle geometries, such as GPLs, and it is adopted herein. In this model, the effective material properties of composites are predicted by incorporating the contributions of the axial (A) and transverse (T) moduli of GPL nanostructures (Wattanasakulpong, 2025). The corresponding expressions are given as follows

$$E_A^f = \frac{(1 + 2\xi_A \eta_A V_{GPL}^f)}{(1 - \eta_A V_{GPL}^f)} E_M, \quad E_T^f = \frac{(1 + 2\xi_T \eta_T V_{GPL}^f)}{(1 - \eta_T V_{GPL}^f)} E_M, \quad (3)$$

$$\xi_A = \frac{l_{GPL}}{h_{GPL}}, \quad \eta_A = \frac{\left(\frac{E_{GPL}}{E_M} - 1 \right)}{\left(\frac{E_{GPL}}{E_M} + 2\xi_A \right)}, \quad \xi_T = \frac{w_{GPL}}{h_{GPL}}, \quad \eta_T = \frac{\left(\frac{E_{GPL}}{E_M} - 1 \right)}{\left(\frac{E_{GPL}}{E_M} + 2\xi_T \right)}.$$

The shear modulus of the face layers, $G(z)$, can be correspondingly determined as follows

$$G^f(z) = \frac{E^f(z)}{2(1 + \nu^f(z))}. \quad (4)$$

The effective mechanical properties of the FGP core are evaluated using the formulation presented by Wattanasakulpong (2025)

$$E^C(z) = E_M \left(1 - e_0 \cos \left(\frac{\pi z}{h_c} \right) \right),$$

$$\nu^C(z) = 0.221 \left(1 - \frac{\rho^C(z)}{\rho_M} \right) + \nu_M \left[0.342 \left(1 - \frac{\rho^C(z)}{\rho_M} \right)^2 - 1.21 \left(1 - \frac{\rho^C(z)}{\rho_M} \right) + 1 \right], \quad (5)$$

$$G^C(z) = \frac{E^C(z)}{2(1 + \nu^C(z))}, \quad \rho^C(z) = \rho_M \left(1 - e_m \cos \left(\frac{\pi z}{h_c} \right) \right),$$

where the superscript “C” denotes the core layer; e_0 is the porosity coefficient associated with stiffness, while e_m represents the porosity coefficient related to mass, defined as follows

$$e_m(z) = \frac{1.121 \left\{ 1 - (1 - e_0 \cos(\pi z/h_c))^{1/2.3} \right\}}{\cos(\pi z/h_c)}. \quad (6)$$

It is noted that $0 \leq e_0 < 1$.

2.2. Beam element model

A generic beam element with length l_e representing the frame structure, as depicted in Fig. 1(a), is considered. The associated degrees of freedom are defined in two Cartesian coordinate systems: the local and the global coordinate systems. In the undeformed configuration, the element is assumed to be inclined at an angle α with respect to the x_g -axis. The global Cartesian coordinate system remains fixed, whereas the local coordinate system is defined such that the x -axis lies along the beam mid-plane and the z -axis is oriented in the thickness direction. Under this assumption, the bending–torsion coupling effect can be neglected.

Following Kosmatka (1995), the displacement components $u(x, z)$ and $w(x, z)$ in the x - and z -directions, respectively, of an arbitrary point with respect to the local Cartesian system are expressed as follows

$$\begin{cases} u(x, z) = u_0(x) - z\theta(x), \\ w(x, z) = w_0(x), \end{cases}, \quad (7)$$

where $u_0(x)$, $w_0(x)$, and $\theta(x)$ denote the local axial displacement, transverse displacement, and cross-section rotation of a point located at coordinate x along the beam axis, respectively.

Let u_0^g , w_0^g , and θ^g denote the global axial displacement in the x_g -direction, the global transverse displacement in the z_g -direction, and the global cross-sectional rotation, respectively. The relationship between the local and global kinematic variables can therefore be expressed as follows

$$\begin{Bmatrix} u_0 \\ w_0 \\ \theta \end{Bmatrix} = \begin{bmatrix} \text{ca} & \text{sa} & 0 \\ -\text{sa} & \text{ca} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{Bmatrix} u_0^g \\ w_0^g \\ \theta^g \end{Bmatrix}, \quad (8)$$

where

$$\text{ca} = \cos \alpha, \quad \text{sa} = \sin \alpha. \quad (9)$$

Based on the von Kármán nonlinear strain–displacement assumption, Eq. (7) yields the axial normal strain ε_{xx} and the transverse shear strain γ_{xz} in the following form

$$\varepsilon_{xx} = \frac{\partial u}{\partial x} + \frac{1}{2} \left(\frac{\partial w}{\partial x} \right)^2 = \varepsilon_m - z \frac{\partial \theta}{\partial x}, \quad \gamma_{xz} = \frac{\partial u}{\partial z} + \frac{\partial w}{\partial x} = \frac{\partial w_0}{\partial x} - \theta, \quad (10)$$

with

$$\varepsilon_m = \frac{\partial u_0}{\partial x} + \frac{1}{2} \left(\frac{\partial w_0}{\partial x} \right)^2, \quad (11)$$

which represents membrane strain. For a linearly elastic material governed by Hooke’s law, the normal and shear stresses, σ_{xx} and τ_{xz} , are expressed in terms of the associated strain components as follows

$$\begin{Bmatrix} \sigma_{xx} \\ \tau_{xz} \end{Bmatrix}^f = \begin{bmatrix} C_{11}^f & 0 \\ 0 & \psi C_{22}^f \end{bmatrix} \begin{Bmatrix} \varepsilon_{xx} \\ \gamma_{xz} \end{Bmatrix}^f, \quad (12)$$

where ψ is the shear correction factor, which is commonly taken as 5/6 for rectangular beam cross-sections, and

$$C_{11}^f = \frac{E^f(z)}{1 - (\nu^f(z))^2}, \quad C_{22}^f = G^f(z) = \frac{E^f(z)}{2(1 + \nu^f(z))}, \quad (13)$$

with f -T, f -C, and f -B correspond to the top layer, core layer, and bottom layer, respectively.

According to the beam theory employed in this study, the elastic strain energy of a beam element is expressed as follows

$$U^e = \frac{1}{2} \int_{V^e} (\sigma_{xx} \varepsilon_{xx} + \tau_{xz} \gamma_{xz}) dV, \quad (14)$$

where V^e represents the volume of the beam element.

By substituting Eqs. (10) and (12) into Eq. (14), the elastic strain energy of the beam element can be reformulated as follows

$$U^e = \frac{1}{2} \int_0^{l_e} \left\{ A_{11} \varepsilon_m^2 + A_{22} \left(\frac{\partial \theta}{\partial x} \right)^2 + A_{33} \left(\frac{\partial w_0}{\partial x} - \theta \right)^2 \right\} dV, \quad (15)$$

where A_{jj} ($j = 1, 2, 3$) represent the stiffness coefficients, which are defined as follows

$$\{A_{11}, A_{22}\} = b \sum_{j=1}^3 \int_{h_{j-1}}^{h_j} \{1, z^2\} C_{11}(z) dz, \quad A_{33} = b \sum_{j=1}^3 \int_{h_{j-1}}^{h_j} \psi C_{22}(z) dz, \quad (16)$$

where $h_0 = -h/2$, $h_3 = h/2$, $h_1 = -h_c/2$, and $h_2 = h_c/2$ are, respectively, the vertical coordinates of the bottommost, topmost surfaces, and the interfaces between the layers. It is noted that the beam element considered herein is symmetric with respect to the mid-plane, and thus, the axial stretching-bending coupling terms do not appear in the strain energy.

The virtual work associated with the external loads applied to the beam element under consideration can be expressed as follows

$$W_{ex}^e = \int_0^{l_e} w(x) q(x) dx, \quad (17)$$

where $q(x)$ denotes the distributed load along the length of the beam element; concentrated loads or moments can be considered as special cases of this loading.

The governing system of nonlinear equations for the frame structure is established based on the following principle of virtual work

$$\sum_{n \in \text{ELE}} (\delta U^e - \delta W_{ex}^e) = 0. \quad (18)$$

Eq. (18) results in a set of coupled nonlinear equations with three primary unknown fields, namely u_0 , w_0 , and θ . Due to the strong nonlinearity of the system, obtaining a closed-form analytical solution is generally intractable. Therefore, the finite element method is employed in this study to formulate the discretized equilibrium equations and to evaluate the structural response of functionally graded porous-core sandwich frames with FG-GPL-reinforced composite face sheets.

3. NUMERICAL PROCEDURE

In the present study, the sandwich frame is discretized into multiple two-node beam elements. For each element, the vector of local degrees of freedom (d.o.f) $\mathbf{d}_e$ is defined in the local Cartesian coordinate system. It consists of the nodal translational displacements together with the cross-sectional rotation, as given below

$$\mathbf{d}_e = \{\mathbf{d}_a \quad \mathbf{d}_b\}^T, \quad (19)$$

in which

$$\mathbf{d}_a = \{u_{01} \ u_{02}\}^T, \quad \mathbf{d}_b = \{w_{01} \ \theta_1 \ w_{02} \ \theta_2\}^T, \quad (20)$$

are the vectors of local axial and bending displacement degrees of freedom, respectively. For $i = 1, 2$, u_{0i} , w_{0i} , and θ_i denote the local axial displacement, transverse displacement, and rotational degree of freedom at node i , respectively (see Fig. 1). In the preceding expressions, the superscript “T” indicates the transpose of a vector or matrix.

The local displacement and rotational fields within the element are interpolated from the corresponding nodal degrees of freedom using the following approximation scheme

$$u_0 = \mathbf{N}\mathbf{d}_a, \quad w_0 = \mathbf{H}_w\mathbf{d}_b, \quad \theta = \mathbf{H}_\theta\mathbf{d}_b, \quad (21)$$

where $\mathbf{N}$, $\mathbf{H}_w$, and $\mathbf{H}_\theta$ denote the interpolation matrices corresponding to u_0 , w_0 , and θ , respectively. Linear shape functions are adopted for the axial displacement u_0 , whereas quadratic and cubic polynomial functions given by Kosmatka (1995), $\mathbf{H}_\theta$ and $\mathbf{H}_w$, respectively, are employed for the rotation θ and transverse displacement w_0 . These shape functions help prevent the derived beam element from suffering from the shear-locking problem.

The linear functions for u_0 are

$$\mathbf{N} = \left[1 - \frac{x}{l_e} \quad \frac{x}{l_e} \right]. \quad (22)$$

And the explicit expressions of $\mathbf{H}_\theta$ and $\mathbf{H}_w$ are given by Kosmatka (1995), and they have the following forms

$$\mathbf{H}_w = \begin{bmatrix} \frac{1}{(1+\phi)} \left\{ 2\left(\frac{x}{l_e}\right)^3 - 3\left(\frac{x}{l_e}\right)^2 - \phi\left(\frac{x}{l_e}\right) + (1+\phi) \right\} \\ \frac{l_e}{(1+\phi)} \left\{ \left(\frac{x}{l_e}\right)^3 - \left(2 + \frac{\phi}{2}\right)\left(\frac{x}{l_e}\right)^2 + \left(1 + \frac{\phi}{2}\right)\left(\frac{x}{l_e}\right) \right\} \\ -\frac{1}{(1+\phi)} \left\{ 2\left(\frac{x}{l_e}\right)^3 - 3\left(\frac{x}{l_e}\right)^2 - \phi\left(\frac{x}{l_e}\right) \right\} \\ \frac{l_e}{(1+\phi)} \left\{ \left(\frac{x}{l_e}\right)^3 - \left(1 - \frac{\phi}{2}\right)\left(\frac{x}{l_e}\right)^2 - \frac{\phi}{2}\left(\frac{x}{l_e}\right) \right\} \end{bmatrix}^T, \quad (23)$$

and

$$\mathbf{H}_\theta = \begin{bmatrix} \frac{6}{(1+\phi)l_e} \left\{ \left(\frac{x}{l_e}\right)^2 - \left(\frac{x}{l_e}\right) \right\} \\ \frac{1}{(1+\phi)} \left\{ 3\left(\frac{x}{l_e}\right)^2 - (4+\phi)\left(\frac{x}{l_e}\right) + (1+\phi) \right\} \\ -\frac{6}{(1+\phi)l_e} \left\{ \left(\frac{x}{l_e}\right)^2 - \left(\frac{x}{l_e}\right) \right\} \\ \frac{1}{(1+\phi)} \left\{ 3\left(\frac{x}{l_e}\right)^2 - (2-\phi)\left(\frac{x}{l_e}\right) \right\} \end{bmatrix}^T, \quad (24)$$

with $\phi = \frac{12}{l_e} \frac{A_{22}}{\psi A_{33}}$ is the ratio of the beam bending stiffness to the shear stiffness; A_{22} and A_{33} are previously defined in Eq. (16).

Differentiating u_0 , w_0 , and θ with respect to x in Eq. (21) yields the following expression

$$\frac{\partial u_0}{\partial x} = \frac{\partial \mathbf{N}}{\partial x} \mathbf{d}_a, \quad \frac{\partial w_0}{\partial x} = \frac{\partial \mathbf{H}_w}{\partial x} \mathbf{d}_b, \quad \frac{\partial \theta}{\partial x} = \frac{\partial \mathbf{H}_\theta}{\partial x} \mathbf{d}_b. \quad (25)$$

The axial strain ε_m given in Eq. (11), together with the interpolation functions in Eqs. (21) and (25), cannot be directly employed in the finite element formulation due to the occurrence of membrane locking. To address this issue, the membrane strain term in Eq. (11) is replaced by an effective strain defined by Crisfield (1991) and combined with the interpolation expression in Eq. (25), yielding

$$\varepsilon_{\text{eff}} = \frac{1}{l_e} \int_0^{l_e} \varepsilon_m dx = \frac{1}{l_e} \int_0^{l_e} \left\{ \frac{\partial \mathbf{N}}{\partial x} \mathbf{d}_a + \frac{1}{2} \left(\frac{\partial \mathbf{H}_w}{\partial x} \mathbf{d}_b \right)^2 \right\} dx. \quad (26)$$

By substituting the interpolation functions in Eqs. (21) and (25) and replacing ε_m with ε_{eff} in Eq. (26), the strain energy of the beam element in Eq. (15) can be obtained in matrix form

$$U^e = \frac{1}{2} \int_0^{l_e} \left\{ A_{11} \varepsilon_{\text{eff}}^2 + A_{22} \left(\frac{\partial \mathbf{H}_\theta}{\partial x} \mathbf{d}_b \right)^2 + A_{33} \left(\frac{\partial \mathbf{H}_w}{\partial x} \mathbf{d}_b - \mathbf{H}_w \mathbf{d}_\theta \right)^2 \right\} dV, \quad (27)$$

and similarly for the external work in Eq. (17)

$$W_{\text{ex}}^e = \int_0^{l_e} (\mathbf{H}_w \mathbf{d}_b) q(x) dx. \quad (28)$$

The internal force vector $\mathbf{f}_{\text{in}}^e$ is obtained by differentiating the elastic strain energy in Eq. (27) with respect to the vector of nodal degrees of freedom, as follows

$$\mathbf{f}_{\text{in}}^e = \frac{\partial U^e}{\partial \mathbf{d}_e} = \left\{ \mathbf{f}_{\text{in}}^a \quad \mathbf{f}_{\text{in}}^b \right\}^T, \quad (29)$$

(6×1)

where

$$\mathbf{f}_{\text{in}}^a = \frac{\partial U^e}{\partial \mathbf{d}_a}, \quad \mathbf{f}_{\text{in}}^b = \frac{\partial U^e}{\partial \mathbf{d}_b}. \quad (30)$$

(2×1) (4×1)

The element vector of external forces, $\mathbf{f}_{\text{ex}}^e$, can be determined from the external work in Eq. (28) as follows

$$\mathbf{f}_{\text{ex}}^e = \{0 \quad 0 \quad \mathbf{H}_w\}^T. \quad (31)$$

(6×6)

The local vectors of d.o.f., internal forces, and external forces are related to the global ones through a transformation matrix, for instance

$$\mathbf{d}_e = \mathbf{T}_t \mathbf{d}_e^g, \quad (32)$$

where $\mathbf{T}_t$ is the transformation matrix expanded from Eq. (8) and has the form

$$\mathbf{T}_t = \begin{bmatrix} \text{ca} & 0 & \text{sa} & 0 & 0 & 0 \\ 0 & \text{ca} & 0 & 0 & \text{sa} & 0 \\ -\text{sa} & 0 & \text{ca} & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & -\text{sa} & 0 & 0 & \text{ca} & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}. \quad (33)$$

Assembling the element internal and external force vectors leads to the nonlinear equilibrium equation of the structure, which can be written in the form given by Crisfield (1991)

$$\mathbf{R}^g(\mathbf{D}^g, \mathbf{F}_{\text{ex}}^g) = \mathbf{F}_{\text{ex}}^g - \mathbf{F}_{\text{in}}^g(\mathbf{D}^g) = 0, \quad (34)$$

where $\mathbf{D}^g$ denotes the global structural vector of degrees of freedom, assembled from the global element vector $\mathbf{d}_e^g$; $\mathbf{F}_{\text{ex}}^g$, $\mathbf{F}_{\text{in}}^g$, and $\mathbf{R}^g$ represent the global vectors of external forces, internal forces, and residual forces, respectively.

The global tangent stiffness matrix $\mathbf{K}_t^g$ is obtained from the incremental variation of the global internal force vector as follows

$$\mathbf{K}_t^g(\mathbf{D}^g) = \frac{\partial \mathbf{F}_{\text{in}}^g(\mathbf{D}^g)}{\partial (\mathbf{D}^g)}. \quad (35)$$

The global internal force vector and the tangent stiffness matrix are obtained from their corresponding element quantities using the relations given in Eqs. (8) and (32), as follows

$$\mathbf{F}_{\text{in}}^g = \sum \mathbf{T}_t^T \mathbf{f}_{\text{in}}^e, \quad \mathbf{K}_t^g = \sum \mathbf{T}_t^T \mathbf{k}_t^e \mathbf{T}_t. \quad (36)$$

Accordingly, the element tangent stiffness matrix can be written in the following form

$$\mathbf{k}_t^e = \frac{\partial \mathbf{f}_{\text{in}}^e}{\partial \mathbf{d}_e} = \begin{bmatrix} \mathbf{k}_t^a & \mathbf{k}_t^{ab} \\ \left(\mathbf{k}_t^{ab}\right)^T & \mathbf{k}_t^b \end{bmatrix}. \quad (37)$$

The submatrices involved in the preceding equation are given by

$$\mathbf{k}_t^a = \frac{\partial \mathbf{f}_{\text{in}}^a}{\partial \mathbf{d}_a}, \quad \mathbf{k}_t^b = \frac{\partial \mathbf{f}_{\text{in}}^b}{\partial \mathbf{d}_b}, \quad \mathbf{k}_t^{ab} = \frac{\partial \mathbf{f}_{\text{in}}^a}{\partial \mathbf{d}_b}. \quad (38)$$

The global tangent stiffness matrix is incorporated into the following equilibrium equation

$$\mathbf{K}_t^g \delta \mathbf{D}^g = \mathbf{R}^g(\mathbf{D}^g, \mathbf{F}_{\text{ex}}^g) = \mathbf{F}_{\text{ex}}^g - \mathbf{F}_{\text{in}}^g(\mathbf{D}^g). \quad (39)$$

The nonlinear equation presented above can be solved for the global vector of degrees of freedom using the Newton–Raphson method. A detailed description of this procedure and its implementation can be found in Crisfield (1991).

Convergence of the iterative process is evaluated using the Euclidean norm of the residual force vector as follows

$$\|\mathbf{R}^g\| \leq \varepsilon_{\text{tol}} \|\mathbf{F}_{\text{ex}}^g\|, \quad (40)$$

where the tolerance ε_{tol} is taken as 10^{-4} in the present study.

4. NUMERICAL ANALYSIS

Three sandwich frames consisting of an FGP core and FG-GPL-reinforced composite face layers are considered in this study. The applied loads, geometric parameters, and selected points for deflection evaluation are illustrated in Fig. 2 following Hsiao and Hou (1987). The details of their cross-sections are shown in Fig. 1(b), and the material properties of copper and graphene platelets adopted from Wattanasakulpong (2025) are presented in Table 2. The displacement and cross-sectional rotational boundary conditions of the frames shown in Fig. 2 are defined as follows:

- Lee's frame (Fig. 2(a)): $u_0^g = w_0^g = 0$ at both ends.
- Portal frame (Fig. 2(b)): $u_0^g = w_0^g = \theta^g = 0$ at both ends.
- Deep arch (Fig. 2(c)): $u_0^g = w_0^g = 0$ at the left end, while $u_0^g = w_0^g = \theta^g = 0$ at the right end.

For convenience in the subsequent discussion, the following dimensionless parameters for the displacements and applied loads are introduced

$$u_0^* = \frac{u_0^g(\text{con.})}{h}, \quad w_0^* = \frac{w_0^g(\text{con.})}{h}, \quad P^* = \frac{PL^2}{E_M I}, \quad (41)$$

where $u_0^g(\text{con.})$ and $w_0^g(\text{con.})$ denote the axial and transverse displacements at the considered point, respectively, and $I = bh^3/12$ is the moment of inertia of the cross-section.

Cross-section as in Fig. 1(b) with $h = 0.01$ m, $b = 0.01$ m, $L = 1$ m, and $R = 0.8$ m

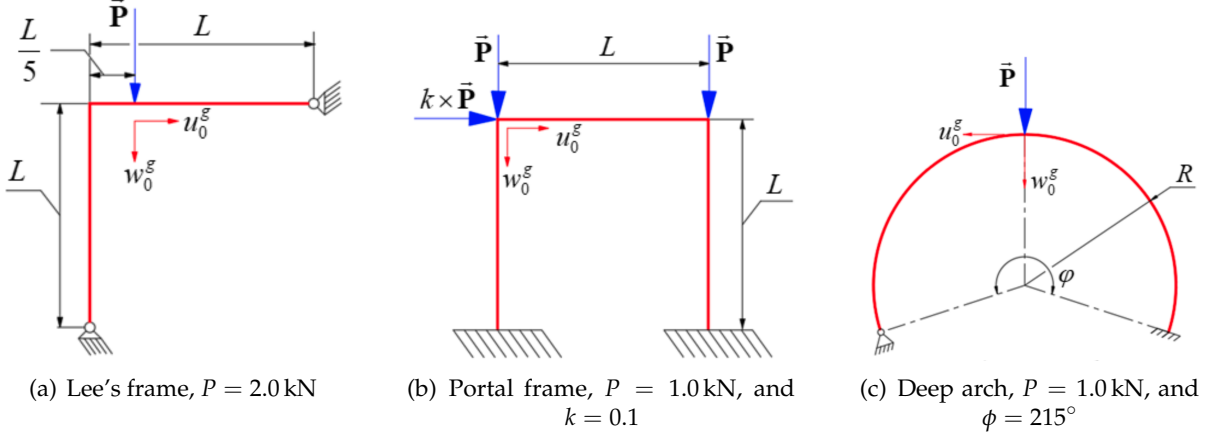


Fig. 2. Frames with geometric data used in the analysis, including the applied loads and considered points for deflection evaluation

Table 2. Material properties of the considered frames

E_M (GPa)	ρ_M (kg/m ³)	ν_M	E_{GPL} (GPa)	ρ_{GPL} (kg/m ³)	ν_{GPL}	l_{GPL} (μ m)	w_{GPL} (μ m)	h_{GPL} (nm)
130	8960	0.34	1010	1062.5	0.186	2.5	1.5	1.5

4.1. Verification of accuracy and convergence

Since no published studies are currently available on the nonlinear bending behavior of frame structures with an FGP core and FG-GPL-reinforced composite face layers, except for the nonlinear bending analysis of beams with an FGP core and FG-GPL-reinforced face layers reported by Wattanasakulpong (2025), the present study validates the proposed finite element model and nonlinear bending algorithm by comparing the obtained results with those reported by Wattanasakulpong (2025). The comparison results, including the linear and nonlinear deflections at the beam midspan, are presented in Table 3. It is evident that the results obtained in this study are in excellent agreement with those reported by Wattanasakulpong (2025), with a maximum relative error of less than 0.15%.

It should be noted that the beam used for comparison with Wattanasakulpong (2025) has the following properties: beam length $L = 1$ m, beam height $h = L/30$, beam thickness $b = h$, porosity coefficient $e_0 = 0.5$, core-to-face thickness ratio $h_c/h_f = 1$, and graphene platelet weight fraction $g_{GPL} = 1.0\%$. The beam is pinned-pinned at both ends and subjected to a uniformly distributed load $q_0 = 1$ kN/m. The dimensionless deflection values and the corresponding errors reported in Table 3 and hereafter are defined according to Wattanasakulpong (2025)

$$\bar{w} = 100 \cdot w_{\max} \cdot \frac{E_M b h^3}{q_0 L^4}, \quad \text{Er}(\%) = \frac{|\text{Present results} - \text{Ref.}|}{\text{Ref.}} \cdot 100\%. \quad (42)$$

Regarding convergence, as shown in Table 3, only 6 elements are sufficient for the linear deflection, whereas 10 elements are required for the nonlinear deflection. Therefore, 10 elements per beam member of the frames are used in all cases, except for the deep arch, for which 30 elements are used.

Table 3. Convergence and comparison of the dimensionless central deflection $\bar{w}$ of a PP sandwich beam composed of an FGP core and two FG-GPL composite face layers under uniform distributed loading

nELE		6	8	10	12	14	16	18	Wattanasakulpong (2025)	Er(%)
FG-X	$\bar{w}_L$	9.1447	9.1447	9.1447	9.1447	9.1447	9.1447	9.1447	9.1498	0.06
	$\bar{w}_{NL}$	8.5964	8.5957	8.5954	8.5954	8.5954	8.5954	8.5954	8.5995	0.05
FG-O	$\bar{w}_L$	9.4107	9.4107	9.4107	9.4107	9.4107	9.4107	9.4107	9.4191	0.09
	$\bar{w}_{NL}$	8.8031	8.8030	8.8028	8.8028	8.8028	8.8028	8.8028	8.8092	0.07
FG-V	$\bar{w}_L$	8.4604	8.4604	8.4604	8.4604	8.4604	8.4604	8.4604	8.4707	0.12
	$\bar{w}_{NL}$	8.0439	8.0438	8.0436	8.0436	8.0436	8.0436	8.0436	8.0520	0.10

Table 4 compares the dimensionless deflection w_0^* of the portal sandwich frame with an FGP core and metallic face sheets obtained by the present formulation with the results of Pham and Le (2025) for $h_c/h_f = 8$. The table shows strong agreement with the reference data, with a maximum relative error of less than 1.4%. It is noted that the third-order shear deformable beam theory is used by Pham and Le (2025), which may explain the slight discrepancies between the results.

Table 4. Comparison of the dimensionless deflections of a portal sandwich frame composed of an FGP core and metallic face layers under three concentrated loads (as in Fig. 2(b), but $h = b = 0.05$ m, $k = 0.5$)

e_0	Sources	$P^* = 0.5$	$P^* = 1.0$	$P^* = 1.5$	$P^* = 2.0$	$P^* = 2.5$	$P^* = 3.0$	$P^* = 3.5$	$P^* = 4.0$
0.2	Pham and Le (2025)	0.0041	0.0145	0.0344	0.0685	0.1239	0.2125	0.3538	0.5819
	Present	0.0041	0.0146	0.0346	0.0688	0.1246	0.2139	0.3563	0.5866
	Er(%)	0.50	0.49	0.51	0.54	0.59	0.64	0.71	0.80
0.4	Pham and Le (2025)	0.0045	0.0160	0.0382	0.0765	0.1397	0.2424	0.4098	0.6870
	Present	0.0045	0.0162	0.0385	0.0771	0.1410	0.2449	0.4145	0.6959
	Er(%)	0.84	0.80	0.82	0.86	0.93	1.02	1.14	1.29
0.6	Pham and Le (2025)	0.0050	0.0179	0.0427	0.0861	0.1590	0.2798	0.4815	0.8254
	Present	0.0051	0.0180	0.0431	0.0869	0.1606	0.2829	0.4874	0.8370
	Er(%)	0.98	0.89	0.89	0.93	1.00	1.09	1.23	1.40
0.8	Pham and Le (2025)	0.0057	0.0201	0.0483	0.0982	0.1835	0.3282	0.5768	1.0166
	Present	0.0057	0.0203	0.0485	0.0987	0.1845	0.3302	0.5805	1.0239
	Er(%)	0.83	0.63	0.56	0.54	0.55	0.58	0.64	0.73

4.2. Numerical examples

4.2.1. Lee's frame

The nonlinear bending behavior of a sandwich Lee's frame (Fig. 2(a)), which consists of a functionally graded porous (FGP) core and two graphene-reinforced face sheets and is subjected to a concentrated load, is investigated. Fig. 3(a) illustrates the deformed configurations of the frame under compressive load levels of 0.4, 0.8, 1.2, 1.6, and 2.0 kN, while the load–deflection curves in Fig. 3(b) clearly demonstrate the nonlinear characteristics of the problem. It should be noted that u_0^* and w_0^* are the dimensionless counterparts of the actual displacements u_0 and w_0 defined in Fig. 2(a).

Fig. 4 illustrates the relationship between the nonlinear dimensionless displacements of Lee's frame and the core-to-face thickness ratio. As shown, when the core-to-face thickness ratio increases, the influence of graphene diminishes, leading to larger nonlinear dimensionless displacements.

Table 5 presents the dimensionless nonlinear displacements (u_0^* and w_0^*) for various values of the porosity coefficient (e_0), GPL distribution patterns (FG-O, FG-X, and FG-V), and the GPL weight fraction (g_{GPL}). From Table 5, it is evident that the FG-O distribution exhibits the largest

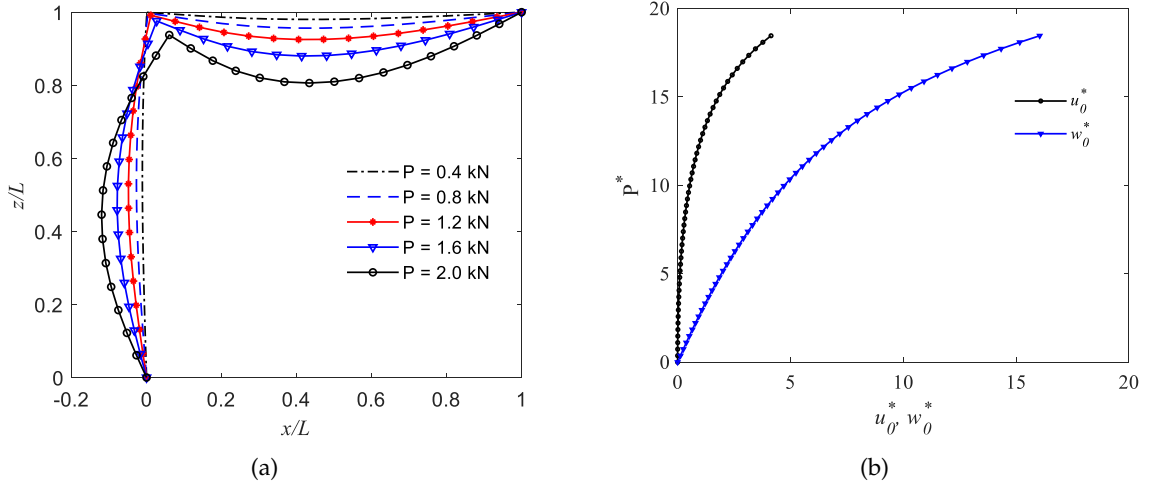


Fig. 3. Deformed configurations (a) and load-deflection curves (b) of the Lee's frame with FG-V type of GPL distribution, $e_0 = 0.2$, $g_{GPL} = 0.5\%$, $h_c/h_f = 1$

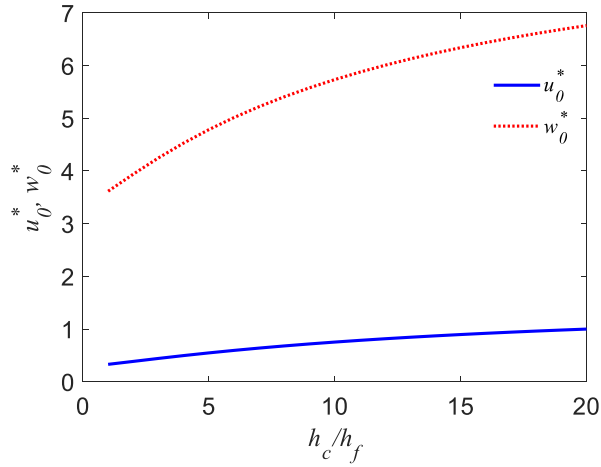


Fig. 4. Nonlinear dimensionless displacements of Lee's frame versus the core-to-face thickness ratio ($e_0 = 0.4$, $g_{GPL} = 0.75\%$, $P = 1.0$ kN, FG-V type)

Table 5. Dimensionless nonlinear displacement responses u_0^* and w_0^* of the sandwich Lee's frame considering different e_0 , g_{GPL} , and GPL distribution types under a concentrated load ($h_c/h_f = 2$)

e_0	Type	$g_{GPL} = 0.25\%$		$g_{GPL} = 0.5\%$		$g_{GPL} = 0.75\%$		$g_{GPL} = 1.0\%$		$g_{GPL} = 2.0\%$	
		u_0^*	w_0^*	u_0^*	w_0^*	u_0^*	w_0^*	u_0^*	w_0^*	u_0^*	w_0^*
0.2	FG-O	12.6996	31.2750	6.4619	21.0679	4.1975	16.1845	3.0165	13.1960	1.2935	7.8744
	FG-X	12.1653	30.5283	6.1942	20.5356	4.0083	15.7306	2.8744	12.8085	1.2311	7.6460
	FG-V	10.3001	27.7617	5.1555	18.3639	3.2595	13.8440	2.3015	11.1687	0.9564	6.5891
0.4	FG-O	13.9332	32.9305	6.7518	21.6334	4.3401	16.5215	3.1005	13.4225	1.3153	7.9539
	FG-X	13.2461	32.0201	6.4640	21.0726	4.1407	16.0495	2.9522	13.0221	1.2513	7.7208
	FG-V	11.0064	28.8408	5.3545	18.7945	3.3547	14.0928	2.3561	11.3314	0.9700	6.6441
0.6	FG-O	15.7476	35.2015	7.0596	22.2212	4.4882	16.8668	3.1870	13.6530	1.3375	8.0339
	FG-X	14.6454	33.8458	6.7496	21.6296	4.2780	16.3759	3.0322	13.2393	1.2718	7.7960
	FG-V	11.8191	30.0352	5.5629	19.2381	3.4527	14.3465	2.4120	11.4963	0.9837	6.6992

deflection, implying the lowest bending stiffness. Conversely, the FG-V distribution yields the smallest deflection, indicating the highest bending stiffness. The distribution of GPLs near the bottom and top surfaces of the FG-V sandwich beam is the main reason for the enhancement of the bending stiffness, thereby reducing the deflections. Furthermore, an increase in the porosity coefficient (e_0) leads to a reduction in bending stiffness, whereas increasing the GPL weight fraction (g_{GPL}) significantly enhances the bending stiffness of the frame.

4.2.2. Portal frame

The nonlinear bending behavior of the portal frame, geometrically depicted in Fig. 2(b), is investigated under a combined loading condition. The structure, which is pinned at both base supports, is subjected to a primary vertical load P applied at the mid-span of the horizontal beam and at the top-right node, together with a proportional horizontal force $k \times P$ applied at the top-left node (where $P = 1.0$ kN and the scaling factor $k = 0.1$). To evaluate the deformation response, the horizontal (u_0) and vertical (w_0) displacements are monitored at the top-left reference point of the frame.

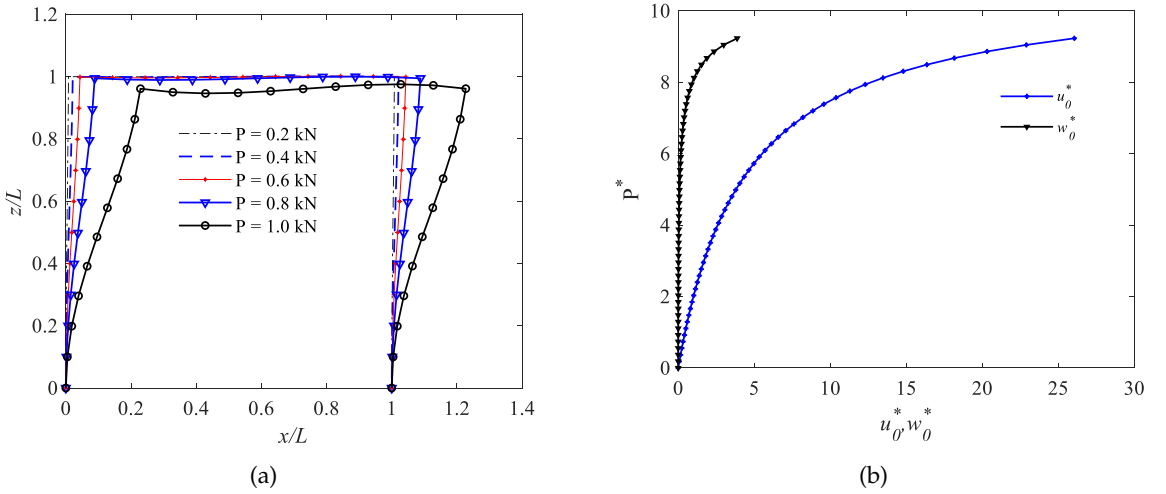


Fig. 5. Deformed configurations (a) and load-deflection curves (b) of the portal frame with FG-X type of GPL distribution, $e_0 = 0.5$, $g_{\text{GPL}} = 2\%$, $h_c/h_f = 10$

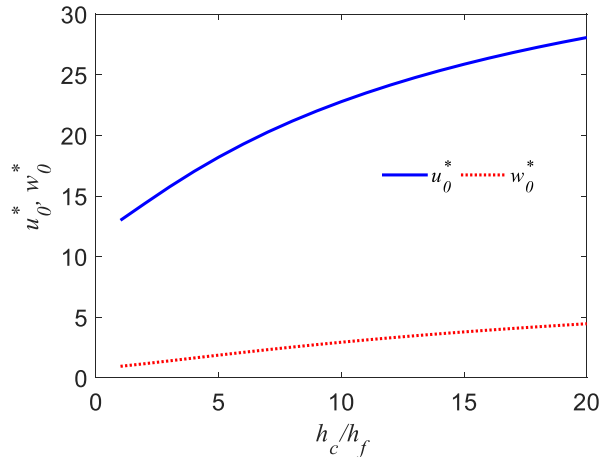


Fig. 6. Nonlinear dimensionless displacements of the portal frame versus the core-to-face thickness ratio, $e_0 = 0.4$, $g_{\text{GPL}} = 0.75\%$, $P = 1.0$ kN, FG-V type

Fig. 5(a) illustrates the progressive deformed configurations of the portal frame at various load increments (ranging from $P = 0.2$ kN to 1.0 kN) for the FG-X GPL distribution pattern, characterized by a porosity coefficient $e_0 = 0.5$, a GPL weight fraction $g_{GPL} = 2\%$, and a core-to-face thickness ratio $h_c/h_f = 10$. As the applied load gradually increases, the frame exhibits a pronounced lateral sway toward the right, accompanied by a noticeable downward deflection of the horizontal beam.

The corresponding load-deflection curves in Fig. 5(b) clearly highlight the highly nonlinear mechanical response of the structure. Notably, the dimensionless horizontal displacement (u_0^*) increases at a significantly faster rate and reaches much larger magnitudes than the vertical displacement (w_0^*) under the same loading conditions.

Fig. 6 presents the variation of the nonlinear dimensionless displacements of the portal frame with the core-to-face thickness ratio. As this ratio increases, the reinforcing effect of graphene becomes less pronounced, resulting in greater nonlinear dimensionless displacements. A similar trend is also observed in Fig. 4.

Table 6. Dimensionless nonlinear displacement responses u_0^* and w_0^* of the sandwich portal frame considering different e_0 , g_{GPL} , and GPL distribution types under a concentrated load ($h_c/h_f = 1$)

e_0	Type	$g_{GPL} = 0.5\%$		$g_{GPL} = 0.75\%$		$g_{GPL} = 1.0\%$		$g_{GPL} = 2.0\%$		$g_{GPL} = 3.0\%$	
		u_0^*	w_0^*	u_0^*	w_0^*	u_0^*	w_0^*	u_0^*	w_0^*	u_0^*	w_0^*
0.2	FG-O	33.3909	6.3559	18.6905	1.9970	13.0411	0.9743	6.1920	0.2221	4.2571	0.1064
	FG-X	29.0306	4.8078	16.5766	1.5719	11.6909	0.7838	5.6436	0.1851	3.9048	0.0900
	FG-V	21.1644	2.5593	12.5565	0.9038	9.0159	0.4678	4.4537	0.1167	3.1004	0.0580
0.4	FG-O	34.0067	6.5920	18.8934	2.0406	13.1404	0.9893	6.2143	0.2238	4.2677	0.1070
	FG-X	29.5060	4.9663	16.7367	1.6025	11.7708	0.7946	5.6622	0.1864	3.9137	0.0905
	FG-V	21.4236	2.6224	12.6487	0.9173	9.0635	0.4729	4.4653	0.1174	3.1060	0.0583
0.6	FG-O	34.6335	6.8367	19.0975	2.0850	13.2397	1.0044	6.2365	0.2255	4.2781	0.1076
	FG-X	29.9888	5.1299	16.8974	1.6335	11.8506	0.8056	5.6806	0.1877	3.9225	0.0910
	FG-V	21.6850	2.6868	12.7408	0.9308	9.1108	0.4779	4.4767	0.1181	3.1115	0.0586

Table 6 provides a quantitative assessment of the dimensionless nonlinear displacement responses (u_0^* and w_0^*) for the sandwich portal frame ($h_c/h_f = 1$), illustrating how different material parameters influence the overall structural stiffness. The results demonstrate that the incorporation of graphene nanoplatelets significantly enhances the stiffness of the frame. As g_{GPL} increases from 0.5% to 3.0%, both the horizontal and vertical displacements decrease markedly across all considered configurations, confirming the high reinforcing efficiency of GPLs in restricting structural deformation. The spatial distribution of GPL reinforcement plays a crucial role in the bending response. Among the three considered functionally graded patterns, the FG-V distribution consistently provides the highest structural stiffness (resulting in the smallest displacements), whereas the FG-O pattern yields the lowest stiffness (resulting in the largest displacements), with the FG-X pattern showing intermediate behavior. For example, at a porosity coefficient of $e_0 = 0.2$ and a GPL weight fraction of 0.5%, the dimensionless horizontal displacement u_0^* for the FG-V distribution is 21.1644, which is significantly smaller than those for the FG-X (29.0306) and FG-O (33.3909) distributions.

4.2.3. Deep arch

The nonlinear bending behavior of the deep arch shown in Fig. 2(c) is investigated under a concentrated vertical load. The arch, with a radius of $R = 0.8$ m and an opening angle of $\phi = 215^\circ$, is pinned at both ends and subjected to a downward point load $\bar{P}$ applied at the apex. The horizontal (u_0) and vertical (w_0) displacements are monitored at this reference point to evaluate the structural response. Fig. 7(a) presents the deformed configurations of the deep arch for load levels ranging from $P = 0.2$ kN to 1.0 kN for the FG-O GPL distribution with

$e_0 = 0.4$, $g_{\text{GPL}} = 0.75\%$, and $h_c/h_f = 6$. As the load increases, the arch undergoes significant downward displacement at the apex accompanied by outward expansion along the curved sides. The load–deflection curves in Fig. 7(b) clearly demonstrate the highly nonlinear response of the structure, indicating a non-proportional relationship between the load parameter P^* and the dimensionless displacements (u_0^* , w_0^*). Fig. 8 also exhibits a trend similar to that observed in Figs. 4 and 6, namely that as the ratio h_c/h_f increases, the stiffness contribution from graphene reinforcement decreases, leading to larger displacements of the frame.

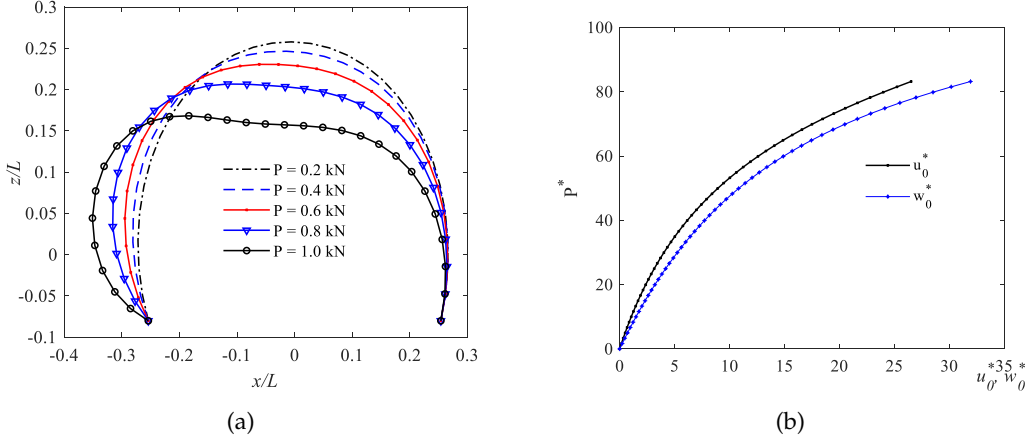


Fig. 7. Deformed configurations (a) and load-deflection curves (b) of the deep arch with FG-O type of GPL distribution, $e_0 = 0.4$, $g_{\text{GPL}} = 0.75\%$, $h_c/h_f = 6$

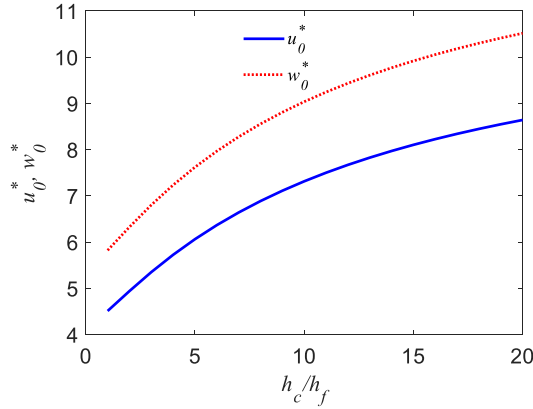


Fig. 8. Nonlinear dimensionless displacements of the deep arch versus the core-to-face thickness ratio, $e_0 = 0.4$, $g_{\text{GPL}} = 0.75\%$, $P = 0.5$ kN, FG-V type

Table 7 summarizes the dimensionless nonlinear displacements (u_0^* , w_0^*) of the sandwich deep arch with $h_c/h_f = 8$, highlighting the influence of key material parameters. Increasing the GPL weight fraction (g_{GPL}) significantly enhances structural stiffness. As g_{GPL} increases from 0.5% to 3.0%, both horizontal and vertical displacements decrease markedly, confirming the strong reinforcing effect of GPLs. For example, in the FG-O configuration with $e_0 = 0.1$, the horizontal displacement u_0^* decreases from 29.7038 to 12.3313. The GPL distribution pattern also affects the bending response. Among the considered patterns, FG-V provides the highest stiffness (smallest displacements), while FG-O results in the lowest stiffness, with FG-X showing intermediate behavior. Finally, the porosity coefficient (e_0) significantly influences the stiffness of the arch. As e_0 increases, the displacements increase due to the weakening effect of internal pores, indicating a reduction in the overall structural rigidity.

Table 7. Dimensionless nonlinear displacement responses u_0^* and w_0^* of the sandwich deep arch considering different e_0 , g_{GPL} , and GPL distribution types under a concentrated load ($h_c/h_f = 8$)

e_0	Type	$g_{GPL} = 0.5\%$		$g_{GPL} = 0.75\%$		$g_{GPL} = 1.0\%$		$g_{GPL} = 2.0\%$		$g_{GPL} = 3.0\%$	
		u_0^*	w_0^*	u_0^*	w_0^*	u_0^*	w_0^*	u_0^*	w_0^*	u_0^*	w_0^*
0.1	FG-O	29.7038	36.4879	26.0113	31.2230	23.0642	27.3102	15.8923	18.6214	12.3313	14.6068
	FG-X	29.6839	36.4583	25.9990	31.2061	23.0627	27.3084	15.9293	18.6638	12.3879	14.6699
	FG-V	29.0707	35.5539	25.2479	30.1866	22.2437	26.2613	15.1027	17.7224	11.6595	13.8606
0.3	FG-O	32.4198	40.6585	28.2791	34.4058	24.9527	29.7912	16.8850	19.7688	12.9486	15.2958
	FG-X	32.3976	40.6233	28.2652	34.3858	24.9511	29.7890	16.9261	19.8166	13.0107	15.3651
	FG-V	31.7131	39.5474	27.4186	33.1798	24.0256	28.5621	16.0169	18.7655	12.2143	14.4775
0.5	FG-O	35.3826	45.5346	30.7844	38.1170	27.0495	32.6622	17.9745	21.0459	13.6147	16.0423
	FG-X	35.3581	45.4929	30.7688	38.0933	27.0477	32.6596	18.0204	21.1001	13.6828	16.1188
	FG-V	34.6029	44.2166	29.8209	36.6647	26.0049	31.2166	17.0069	19.9118	12.8105	15.1425

5. CONCLUSION

This article investigated the nonlinear bending behavior of sandwich frames with a porous core and FG-GPL-reinforced composite face layers. By employing Timoshenko beam theory and von Kármán geometric nonlinearity, a finite beam formulation, in which coupled higher-order shape functions are used, has been derived and used to establish the discretized nonlinear equilibrium equation. Nonlinear responses of various sandwich frames, including the Lee frame, portal frame, and deep arch composed of a porous core and FG-GPL face layers, have been computed using the iterative procedure. The numerical studies demonstrated the high accuracy of the beam model, with maximum discrepancies of only 0.10% for nonlinear deflections in comparison with the results reported in the literature. The convergence of the derived formulation is fast, and accurate deflections can be predicted by using a small number of elements. Comprehensive parametric analyses revealed that the sandwich frames exhibit pronounced geometric nonlinearity and complex equilibrium paths under applied loads. It has been shown that increasing the weight fraction of GPLs was found to be an effective reinforcement mechanism, significantly reducing structural displacements and enhancing the overall stiffness. The spatial distribution of GPLs also plays a crucial role in determining structural rigidity. Among the three types of GPL distribution considered herein, the FG-V distribution consistently yields the highest stiffness, whereas the FG-O pattern results in the most flexible response. Conversely, an increase in the porosity coefficient weakens the material matrix, leading to larger structural deformations and reduced stiffness across all configurations. Overall, these findings provide valuable insights into predicting and optimizing the mechanical behavior of advanced composite frames by tailoring the GPL weight fraction, distribution pattern, and internal porosity level.

DECLARATION OF COMPETING INTEREST

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

CREDIT AUTHOR STATEMENT

Thu Nga Dinh: *Writing – original draft, Visualization, Formal analysis.* Ngoc Duyen Dang: *Data curation, Investigation.* Dinh Kien Nguyen: *Supervision, Writing – review & editing.* Cong Ich Le: *Software, Validation, Conceptualization, Methodology.*

FUNDING

This research received no specific grant from any funding agency in the public, commercial, or not-for-profit sectors.

REFERENCES

- Anirudh, B., Ben Zineb, T., Polit, O., Ganapathi, M., & Prateek, G. (2020). Nonlinear bending of porous curved beams reinforced by functionally graded nanocomposite graphene platelets applying an efficient shear flexible finite element approach. *International Journal of Non-Linear Mechanics*, 119, 103346. <https://doi.org/10.1016/j.ijnonlinmec.2019.103346>
- Arshid, E., Amir, S., & Loghman, A. (2021). Thermal buckling analysis of FG graphene nanoplatelets reinforced porous nanocomposite MCST-based annular/circular microplates. *Aerospace Science and Technology*, 111, 106561. <https://doi.org/10.1016/j.ast.2021.106561>
- Chen, D., Yang, J., & Kitipornchai, S. (2015). Elastic buckling and static bending of shear deformable functionally graded porous beam. *Composite Structures*, 133, 54–61. <https://doi.org/10.1016/j.compstruct.2015.07.052>
- Chen, D., Gao, K., Yang, J., & Zhang, L. (2023). Functionally graded porous structures: Analyses, performances, and applications – a review. *Thin-Walled Structures*, 191, 111046. <https://doi.org/10.1016/j.tws.2023.111046>
- Chen, X., Shen, H.-S., & Li, C. (2023). Re-examination of nonlinear vibration, nonlinear bending and thermal postbuckling of porous sandwich beams reinforced by graphene platelets. *Composite Structures*, 322, 117392. <https://doi.org/10.1016/j.compstruct.2023.117392>
- Crisfield, M. A. (1991). *Non-linear finite element analysis of solids and structures: Volume 1: Essentials*. John Wiley & Sons.
- Dang, V.-H., & Do, Q.-C. (2021). Nonlinear vibration and stability of functionally graded porous microbeam under electrostatic actuation. *Archive of Applied Mechanics*, 91(5), 2301–2329. <https://doi.org/10.1007/s00419-021-01884-7>
- Hsiao, K. M., & Hou, F. Y. (1987). Nonlinear finite element analysis of elastic frames. *Computers & Structures*, 26(4), 693–701. [https://doi.org/10.1016/0045-7949\(87\)90016-2](https://doi.org/10.1016/0045-7949(87)90016-2)
- Karamanli, A., & Vo, T. P. (2021). A quasi-3D theory for functionally graded porous microbeams based on the modified strain gradient theory. *Composite Structures*, 257, 113066. <https://doi.org/10.1016/j.compstruct.2020.113066>
- Kosmatka, J. B. (1995). An improved two-node finite element for stability and natural frequencies of axial-loaded Timoshenko beams. *Computers & Structures*, 57(1), 141–149. [https://doi.org/10.1016/0045-7949\(94\)00595-T](https://doi.org/10.1016/0045-7949(94)00595-T)
- Lin, J., Li, J., Guan, Y., Zhao, G., Naceur, H., & Couellier, D. (2018). Geometrically nonlinear bending analysis of functionally graded beam with variable thickness by a meshless method. *Composite Structures*, 189, 239–246. <https://doi.org/10.1016/j.compstruct.2018.01.063>
- Liu, Z., Yang, C., Gao, W., Wu, D., & Li, G. (2019). Nonlinear behaviour and stability of functionally graded porous arches with graphene platelets reinforcements. *International Journal of Engineering Science*, 137, 37–56. <https://doi.org/10.1016/j.ijengsci.2018.12.003>
- Mirjavadi, S. S., Matin, A., Shafiei, N., Rabby, S., & Mohasel Afshari, B. (2017). Thermal buckling behavior of two-dimensional imperfect functionally graded microscale-tapered porous beam. *Journal of Thermal Stresses*, 40(10), 1201–1214. <https://doi.org/10.1080/01495739.2017.1332962>
- Nguyen, D. K., Nguyen, K. V., Dinh, V. M., Gan, B. S., & Alexandrov, S. (2018). Nonlinear bending of elastoplastic functionally graded ceramic-metal beams subjected to nonuniform distributed loads. *Applied Mathematics and Computation*, 333, 443–459. <https://doi.org/10.1016/j.amc.2018.03.100>

- Niknam, H., Fallah, A., & Aghdam, M. M. (2014). Nonlinear bending of functionally graded tapered beams subjected to thermal and mechanical loading. *International Journal of Non-Linear Mechanics*, 65, 141–147. <https://doi.org/10.1016/j.ijnonlinmec.2014.05.011>
- Pham, V. N., & Le, C. I. (2025). Nonlinear bending of FGP beams and frames based on a third-order shear deformable theory. *Journal of Physics: Conference Series*, 2949(1), 012040. <https://doi.org/10.1088/1742-6596/2949/1/012040>
- Polit, O., Anant, C., Anirudh, B., & Ganapathi, M. (2019). Functionally graded graphene reinforced porous nanocomposite curved beams: Bending and elastic stability using a higher-order model with thickness stretch effect. *Composites Part B: Engineering*, 166, 310–327. <https://doi.org/10.1016/j.compositesb.2018.11.074>
- Sahmani, S., Aghdam, M. M., & Rabczuk, T. (2018). Nonlinear bending of functionally graded porous micro/nano-beams reinforced with graphene platelets based upon nonlocal strain gradient theory. *Composite Structures*, 186, 68–78. <https://doi.org/10.1016/j.compstruct.2017.11.082>
- Shen, H.-S., & Xiang, Y. (2013). Nonlinear analysis of nanotube-reinforced composite beams resting on elastic foundations in thermal environments. *Engineering Structures*, 56, 698–708. <https://doi.org/10.1016/j.engstruct.2013.06.002>
- Sitli, Y., Elmhaia, O., Mesmoudi, S., Askour, O., Rammane, M., Hilali, Y., & Bourihane, O. (2024). Investigating the bending and buckling behaviors of composite porous beams reinforced with carbon nanotubes and graphene platelets using a TRPIM path following mesh-free approach. *Computers & Structures*, 303, 107492. <https://doi.org/10.1016/j.compstruc.2024.107492>
- Songsuwan, W., Prabkeao, C., & Wattanasakulpong, N. (2023). On linear and nonlinear bending of functionally graded graphene nanoplatelet reinforced composite beams using Gram-Schmidt-Ritz method. *Mechanics Based Design of Structures and Machines*, 51(10), 5710–5736. <https://doi.org/10.1080/15397734.2021.2010571>
- Tam, M., Yang, Z., Zhao, S., Zhang, H., Zhang, Y., & Yang, J. (2020). Nonlinear bending of elastically restrained functionally graded graphene nanoplatelet reinforced beams with an open edge crack. *Thin-Walled Structures*, 156, 106972. <https://doi.org/10.1016/j.tws.2020.106972>
- Wang, Y., Ma, H., Xie, K., Fu, T., Chen, J., & Liu, Y. (2022). Nonlinear bending of a sandwich beam with metal foam and GPLRC face-sheets using Chebyshev–Ritz method: Effects of agglomeration and internal pore. *Thin-Walled Structures*, 181, 110035. <https://doi.org/10.1016/j.tws.2022.110035>
- Wattanasakulpong, N. (2025). Nonlinear bending of sandwich beams made of FG-GPLRC faces and FGP core using different micromechanical models for approximating material properties. *International Journal of Solids and Structures*, 317, 113420. <https://doi.org/10.1016/j.ijsolstr.2025.113420>
- Yang, T., Tang, Y., Li, Q., & Yang, X.-D. (2018). Nonlinear bending, buckling and vibration of bi-directional functionally graded nanobeams. *Composite Structures*, 204, 313–319. <https://doi.org/10.1016/j.compstruct.2018.07.045>
- Zghal, S., Ataoui, D., & Dammak, F. (2022). Static bending analysis of beams made of functionally graded porous materials. *Mechanics Based Design of Structures and Machines*, 50(3), 1012–1029. <https://doi.org/10.1080/15397734.2020.1748053>
- Zhao, S., Zhang, Y., Wu, H., Zhang, Y., Yang, J., & Kitipornchai, S. (2022). Tunable nonlinear bending behaviors of functionally graded graphene origami enabled auxetic metamaterial beams. *Composite Structures*, 301, 116222. <https://doi.org/10.1016/j.compstruct.2022.116222>