

# CLOSED-FORM SOLUTIONS FOR LOVE-WAVE PROPAGATION AND EXCITATION IN ORTHOTROPIC LAYERED HALF-SPACES

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**Abstract.** Love waves are horizontally polarized shear waves guided by a surface layer and are highly sensitive to near-surface properties. While their behavior in isotropic media is well established, closed-form solutions for orthotropic layered structures under external excitation remain limited. This study presents closed-form solutions for both propagation and excitation of Love waves in an orthotropic elastic layer bonded to an orthotropic half-space. The dispersion relation is first derived within the framework of anti-plane shear motion using surface and interfacial boundary conditions. The elastodynamic reciprocity theorem is then applied to obtain explicit expressions for modal amplitudes generated by time-harmonic loading. Numerical results illustrate the dispersive characteristics and excitation behavior of the waves, highlighting the dependence of modal amplitudes on the frequency–thickness parameter. The proposed formulation provides a unified and efficient analytical framework for guided-wave analysis in anisotropic layered media.

**Keywords:** Love waves, orthotropic layered half-space, closed-form solution, elastodynamic reciprocity, dispersion analysis.

## 1. INTRODUCTION

Wave propagation in elastic solids is a fundamental topic in elastodynamics and plays a crucial role in many applications, including nondestructive evaluation, geophysics, and structural health monitoring (Achenbach, 1973; Nayfeh, 1995; Rose, 2014). Among guided surface waves, Love waves are of particular interest due to their horizontally polarized shear (SH) motion confined near the surface by an elastic layer bonded to a substrate. Owing to their strong sensitivity to near-surface material properties and interfacial conditions, Love waves have been widely used for material characterization and subsurface investigation in both engineering and geophysical contexts (Ebrahiminejad et al., 2022; Rose, 2014; Tiersten, 1969).

Most classical studies on Love waves are based on the assumption of isotropic elasticity, which enables relatively simple analytical formulations (Nayfeh, 1995; Tiersten, 1969). However, many natural and engineered materials, such as sedimentary rocks, jointed media, and fiber-reinforced composites, exhibit pronounced orthotropic behavior with direction-dependent stiffness properties (Babanouri & Fattahi, 2018; Solie & Auld, 1973). Although anisotropic effects have been extensively investigated for Rayleigh and other guided waves (Kaplunov et al., 2019; Vinh & Ngoc Anh, 2014), closed-form solutions for Love-wave propagation in orthotropic layered half-spaces remain relatively limited due to the increased mathematical complexity. Recent studies have also investigated long-wave approximations of Love waves near cut-off

frequencies (Ege et al., 2026; Erbaş et al., 2024), highlighting asymptotic behaviors of guided waves in layered media.

The analysis becomes even more challenging when wave motion is generated by external sources. Unlike free-wave problems, forced wave analysis involves nonhomogeneous boundary conditions and requires the explicit determination of wave amplitudes in addition to dispersion characteristics (Achenbach, 1973; Rose, 2014). Conventional approaches, such as integral transform methods combined with modal expansions or stiffness-matrix techniques, often result in solutions expressed in integral or matrix form (Lowe, 1995; Vasudevan & Mal, 1985; Xu & Ma, 2022). Consequently, explicit closed-form solutions for forced guided waves in anisotropic layered media are rarely available.

An efficient alternative framework is provided by the elastodynamic reciprocity theorem (Achenbach, 2003), which enables the direct evaluation of wave amplitudes without solving the full boundary-value problem. Reciprocity-based approaches have been successfully applied to a variety of wave propagation problems in layered media, including both isotropic and anisotropic configurations (Kulkarni & Achenbach, 2008; Nguyen & Phan, 2019; Phan et al., 2013; Phan et al., 2019). In particular, recent studies have demonstrated the effectiveness of this method for layered systems under time-harmonic loading, including multi-layered configurations and orthotropic half-spaces (Le et al., 2022; Phan et al., 2019). For example, reciprocity-based formulations have been used to derive analytical solutions for guided waves in layered media with multiple layers and complex boundary conditions, providing explicit expressions for modal amplitudes.

Despite these advances, reciprocity-based analytical solutions for Love waves in orthotropic layered half-space configurations remain largely unexplored. Existing studies have primarily focused on Rayleigh waves or isotropic Love-wave models, and extensions to orthotropic layered systems are still limited. In particular, a unified closed-form solution framework that combines dispersion analysis and forced-wave excitation for Love waves in orthotropic layered media is still lacking.

The present study aims to fill this gap. We develop closed-form solutions for Love-wave propagation and excitation in an orthotropic elastic layer bonded to an orthotropic half-space under time-harmonic shear loading. First, the free Love-wave problem is formulated within the framework of anti-plane shear motion, and the corresponding dispersion relation is derived. Next, the elastodynamic reciprocity theorem is employed to obtain closed-form expressions for the amplitudes of Love-wave modes generated by external loading. The proposed formulation provides clear physical insight into the role of orthotropic shear stiffness parameters and recovers the classical isotropic results as a limiting case. Owing to its analytical and systematic nature, the present approach offers a useful closed-form analytical framework for modeling guided wave phenomena in anisotropic layered media.

## 2. FREE LOVE-WAVE PROPAGATION IN AN ORTHOTROPIC LAYERED HALF-SPACE

We present explicit solutions for Love waves propagating in an orthotropic half-space coated by an orthotropic thin layer. These solutions also serve as a basis for evaluating Love-wave fields generated by time-harmonic sources via the reciprocity theorem in the subsequent section. The governing equations for elastic wave motion in an orthotropic medium are given by Achenbach (1973) and Phan (2020)

$$\tau_{ij,i} = \rho \ddot{u}_j \quad i, j = 1, 2, 3, \quad (1)$$

where  $\tau_{ij}$ ,  $u_j$  and  $\rho$  denote the stress components, displacement components, and mass density, respectively. These equations describe the stress–displacement relations in the absence of body forces.

Love waves correspond to anti-plane shear (SH) motion propagating along the  $x_1$ -direction. In the present work, the coordinate system  $(x_1, x_2, x_3)$  is chosen to coincide with the principal material axes of the orthotropic layer and half-space, and the wave propagation is restricted to the principal direction  $x_1$ . For propagation along a non-principal direction, the anti-plane SH motion is no longer uncoupled from the in-plane components due to the anisotropic stiffness coupling. As a result, the classical Love-wave structure does not strictly hold, and the analysis becomes significantly more involved.

Under this assumption, the displacement field is independent of  $x_3$ , and the out-of-plane displacement  $u_3$  fully characterizes the motion. The constitutive relations then reduce to  $\tau_{13} = C_{55}u_{3,1}$ ,  $\tau_{23} = C_{44}u_{3,2}$ , which leads to the governing wave equation

$$C_{55}u_{3,11} + C_{44}u_{3,22} = \rho u_{3,tt}. \quad (2)$$

We consider a layered configuration consisting of an orthotropic layer  $\mathbf{D}$  of thickness  $h$  perfectly bonded to an orthotropic half-space  $\hat{\mathbf{D}}$  at  $x_2 = 0$ , as illustrated in Fig. 1. Both media are homogeneous, with stiffness matrices  $\mathbf{C}$  and  $\hat{\mathbf{C}}$ , respectively.

In general, guided waves in a layer can be decomposed into six partial waves (three incident and three reflected), whereas only three waves exist in a half-space due to the absence of reflections. Focusing on anti-plane motion, only SH waves are relevant. Accordingly, Love waves in the layer are represented as the superposition of two oppositely propagating SH waves along the thickness direction

$$u_3(x_1, x_2, t) = \left( A_1 e^{ik\beta x_2} + A_2 e^{-ik\beta x_2} \right) e^{ik(x_1 - ct)}. \quad (3)$$

In the half-space, the radiation condition requires decay with depth, yielding

$$\hat{u}_3(x_1, x_2, t) = \hat{A} e^{-k\hat{\beta}x_2} e^{ik(x_1 - ct)}, \quad (4)$$

where  $k$  is the wavenumber in the  $x_1$ -direction,  $i$  is the imaginary unit, and  $c$  denotes the phase velocity, while  $A_1$ ,  $A_2$ , and  $\hat{A}$  are unknown amplitudes. The parameters  $\beta$ ,  $\hat{\beta}$  are defined as

$$\beta^2 = \frac{\rho c^2 - C_{55}}{C_{44}}, \quad \hat{\beta}^2 = \frac{\hat{C}_{55} - \hat{\rho} c^2}{\hat{C}_{44}}. \quad (5)$$

The corresponding shear stress components are obtained from the constitutive relations as

$$\tau_{23} = ik\beta C_{44} \left( A_1 e^{ik\beta x_2} - A_2 e^{-ik\beta x_2} \right) e^{ik(x_1 - ct)}, \quad (6)$$

$$\tau_{13} = ikC_{55} \left( A_1 e^{ik\beta x_2} + A_2 e^{-ik\beta x_2} \right) e^{ik(x_1 - ct)}, \quad (7)$$

$$\hat{\tau}_{23} = -k\hat{\beta}\hat{C}_{44}\hat{A} e^{-k\hat{\beta}x_2} e^{ik(x_1 - ct)}, \quad (8)$$

$$\hat{\tau}_{13} = ik\hat{C}_{55}\hat{A} e^{-k\hat{\beta}x_2} e^{ik(x_1 - ct)}. \quad (9)$$

The boundary conditions consist of a traction-free surface at  $x_2 = -h$  and perfect bonding at the interface  $x_2 = 0$

$$\tau_{23} = 0, \quad \text{at } x_2 = -h, \quad (10)$$

$$u_3 = \hat{u}_3, \tau_{23} = \hat{\tau}_{23}, \quad \text{at } z = 0. \quad (11)$$

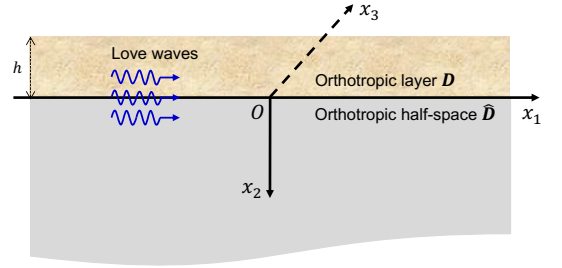


Fig. 1. Love waves in a layered half-space

Substitution of the displacement and stress fields into these conditions yields a homogeneous linear system for the amplitudes, which can be written in matrix form as

$$\begin{bmatrix} e^{-ik\beta h} & -e^{ik\beta h} & 0 \\ 1 & 1 & -1 \\ -\beta C_{44} & \beta C_{44} & i\hat{\beta}\hat{C}_{44} \end{bmatrix} \begin{bmatrix} A_1 \\ A_2 \\ \hat{A} \end{bmatrix} = \mathbf{0}. \quad (12)$$

For nontrivial solutions, the determinant of the three-by-three matrix in Eq. (12) must vanish, leading to the dispersion (characteristic) equation of Love waves in the orthotropic layered half-space

$$\tan(k\beta h) = \frac{\hat{\beta}\hat{C}_{44}}{\beta C_{44}}, \quad (13)$$

This relation depends on the wavenumber  $k$ , indicating that Love waves are dispersive.

Since the system admits infinitely many solutions, the amplitudes can be expressed in terms of a single arbitrary constant  $A$  as  $A_1 = Ad_1$ ,  $A_2 = Ad_2$ ,  $\hat{A} = A\hat{d}$ , where the dimensionless coefficients depend only on material properties and are given by

$$d_1 = e^{ik\beta h}, \quad d_2 = e^{-ik\beta h}, \quad \hat{d} = e^{ik\beta h} + e^{-ik\beta h}. \quad (14)$$

Accordingly, the displacement and shear stress fields reduce to

$$u_3 = A \left( d_1 e^{ik\beta x_2} + d_2 e^{-ik\beta x_2} \right) e^{ik(x_1 - ct)}, \quad (15)$$

$$\tau_{13} = ikC_{55}A \left( d_1 e^{ik\beta x_2} + d_2 e^{-ik\beta x_2} \right) e^{ik(x_1 - ct)}, \quad (16)$$

$$\hat{u}_3 = \hat{d}Ae^{-k\hat{\beta}x_2}e^{ik(x_1 - ct)}, \quad (17)$$

$$\hat{\tau}_{13} = ik\hat{C}_{55}A\hat{d}e^{-k\hat{\beta}x_2}e^{ik(x_1 - ct)}. \quad (18)$$

Thus, the wave field is fully characterized by a single unknown amplitude  $A$ , referred to as the relative amplitude, which will be determined in the next section using the reciprocity theorem.

### 3. RECIPROCITY-BASED FORMULATION FOR LOVE-WAVE EXCITATION

This section derives analytical expressions for Love-wave amplitudes generated by a time-harmonic source in an orthotropic layered half-space. The formulation is based on the elastodynamic reciprocity theorem applied to two admissible states of the medium.

Let state  $\mathcal{A}$  denote the actual state corresponding to the wave field generated by external loading, and state  $\mathcal{B}$  denote a virtual state chosen as a free Love-wave mode propagating in the layered structure. The reciprocity relation between these two states can be written as

$$\begin{aligned} & \int_{\Omega} \left( f_j^{\mathcal{A}} u_j^{\mathcal{B}} - f_j^{\mathcal{B}} u_j^{\mathcal{A}} \right) d\Omega + \int_{\hat{\Omega}} \left( \hat{f}_j^{\mathcal{A}} \hat{u}_j^{\mathcal{B}} - \hat{f}_j^{\mathcal{B}} \hat{u}_j^{\mathcal{A}} \right) d\hat{\Omega} \\ &= \int_S \left( \tau_{ij}^{\mathcal{B}} u_j^{\mathcal{A}} - \tau_{ij}^{\mathcal{A}} u_j^{\mathcal{B}} \right) n_i dS + \int_{\hat{S}} \left( \hat{\tau}_{ij}^{\mathcal{B}} \hat{u}_j^{\mathcal{A}} - \hat{\tau}_{ij}^{\mathcal{A}} \hat{u}_j^{\mathcal{B}} \right) \hat{n}_i d\hat{S}, \end{aligned} \quad (19)$$

where  $S$  and  $\hat{S}$  are external boundaries of the layer and the half-space without the interface, respectively,  $n$  and  $\hat{n}$  are normal vectors, and  $f$  represents the body force, see Fig. 2.

We consider a time-harmonic horizontal line load applied in the  $x_3$ -direction at  $(x_1^0, x_2^0)$  in the form of a Delta function as

$$f^{\mathcal{A}} = R\delta(x_1 - x_1^0)\delta(x_2 - x_2^0)e^{-ikct}. \quad (20)$$

This loading generates Love waves propagating in both the positive and negative  $x_1$ -directions with unknown modal amplitudes  $A_m^+$  and  $A_m^-$ , respectively, where  $m = 0, 1, \dots, \infty$  indicates the mode. This is the actual state  $\mathcal{A}$  whose amplitudes are to be determined. The

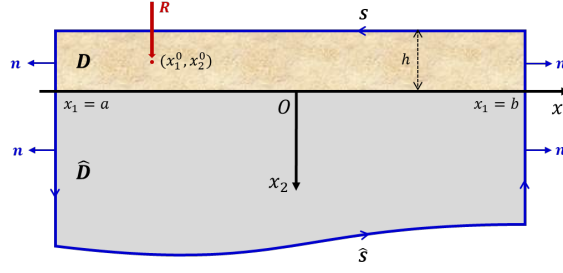


Fig. 2. Layered half-space under a time-harmonic load

expansions for the far-field displacements and the stresses of state  $\mathcal{A}$  in the positive direction ( $x_1 > 0$ ) may be written as

$$u_3 = \sum_{m=0}^{\infty} u_3^m = \sum_{m=0}^{\infty} A_m^+ \left( d_{1m} e^{ik_m \beta x_2} + d_{2m} e^{-ik_m \beta x_2} \right) e^{ik_m (x_1 - c_m t)}, \quad (21)$$

$$\hat{u}_3 = \sum_{m=0}^{\infty} \hat{u}_3^m = \sum_{m=0}^{\infty} A_m^+ \hat{d}_m e^{-k_m \hat{\beta} x_2} e^{ik_m (x_1 - c_m t)}. \quad (22)$$

The tractions on a plane normal to  $x_1$  are

$$\tau_{13} = \sum_{m=0}^{\infty} \tau_{13}^m = \sum_{m=0}^{\infty} ik_m C_{55} A_m^+ \left( d_{1m} e^{ik_m \beta x_2} + d_{2m} e^{-ik_m \beta x_2} \right) e^{ik_m (x_1 - c_m t)}, \quad (23)$$

$$\hat{\tau}_{13} = \sum_{m=0}^{\infty} \hat{\tau}_{13}^m = \sum_{m=0}^{\infty} ik_m \hat{C}_{55} A_m^+ \hat{d}_m e^{-k_m \hat{\beta} x_2} e^{ik_m (x_1 - c_m t)}. \quad (24)$$

For brevity, only the positive-going field in the region  $x_1 > 0$  is written explicitly. The corresponding negative-going field in  $x_1 < 0$  has an analogous form and is therefore omitted. Notably, Eqs. (25)–(28) already represent such a negative-going Love-wave mode (mode  $n$ ) with amplitude  $B_n$ , which is employed as the virtual state in the reciprocity formulation.

We first choose state  $\mathcal{B}$  in the negative  $x_1$ -direction that is a free Love wave of mode  $n$ , which may be expressed as

$$u_3^n = B_n \left( d_{1n} e^{ik_n \beta x_2} + d_{2n} e^{-ik_n \beta x_2} \right) e^{-ik_n (x_1 + c_n t)}, \quad (25)$$

$$\hat{u}_3^n = B_n \hat{d}_n e^{-k_n \hat{\beta} x_2} e^{-ik_n (x_1 + c_n t)}, \quad (26)$$

$$\tau_{13}^n = -ik_n C_{55} B_n \left( d_{1n} e^{ik_n \beta x_2} + d_{2n} e^{-ik_n \beta x_2} \right) e^{-ik_n (x_1 + c_n t)}, \quad (27)$$

$$\hat{\tau}_{13}^n = -ik_n \hat{C}_{55} B_n \hat{d}_n e^{-k_n \hat{\beta} x_2} e^{-ik_n (x_1 + c_n t)} \quad (28)$$

Application of the reciprocity relation is now considered over a domain defined by  $a \leq x_1 \leq b, -h \leq x_2 < \infty$ . There is no contribution of the integral along the top surface of the layered half-space since a free boundary condition is applied. Moreover, the contribution from the integration along the line at a constant  $x_2$  is not included since it vanishes as  $x_2 \rightarrow \infty$ . The boundary integral over  $x_1 = a$  vanishes because the actual state  $\mathcal{A}$  and virtual state  $\mathcal{B}$  correspond to co-propagating modes on this boundary, for which the reciprocity integrand is zero. Therefore, Eq. (19) can be reduced to

$$RB_n \left( d_{1n} e^{ik_n \beta x_2^0} + d_{2n} e^{-ik_n \beta x_2^0} \right) e^{-ik_n x_1^0} e^{-2ik_n c_n t} = \sum_{m=0}^{\infty} \left( \int_{-h}^0 H_{AB}^{mn} |_{x_1=b} dx_2 + \int_0^{\infty} \hat{H}_{AB}^{mn} |_{x_1=b} dx_2 \right), \quad (29)$$

where

$$H_{AB}^{mn} = \tau_{13}^B u_3^A - \tau_{13}^A u_3^B, \quad (30)$$

$$\hat{H}_{AB}^{mn} = \hat{\tau}_{13}^{Bn} \hat{u}_3^{Am} - \hat{\tau}_{13}^{Am} \hat{u}_3^{Bn}. \quad (31)$$

By the orthogonality relation of Le et al. (2022) for counter-propagating Love modes, the right-hand side of Eq. (29) cancels out for  $m \neq n$ . Thus

$$RB_n \left( d_{1n} e^{ik_n \beta x_2^0} + d_{2n} e^{-ik_n \beta x_2^0} \right) e^{-ik_n x_1^0} e^{-2ik_n c_n t} = \int_{-h}^0 H_{AB}^{nn} |_{x_1=b} dx_2 + \int_0^\infty \hat{H}_{AB}^{nn} |_{x_1=b} dx_2, \quad (32)$$

where

$$H_{AB}^{nn} = -2ik_n C_{55} A_n^+ B_n \left( d_{1n} e^{ik_n \beta x_2} + d_{2n} e^{-ik_n \beta x_2} \right)^2 e^{-2ik_n c_n t}, \quad (33)$$

$$\hat{H}_{AB}^{nn} = -2ik_n \hat{C}_{55} A_n^+ B_n \hat{d}_n e^{-2k_n \hat{\beta} x_2} e^{-2ik_n c_n t}. \quad (34)$$

We finally obtain

$$A_n^+ = \frac{R \left( d_{1n} e^{ik_n \beta x_2^0} + d_{2n} e^{-ik_n \beta x_2^0} \right) e^{-ik_n x_1^0}}{2 (C_{55} L_n + \hat{C}_{55} \hat{L}_n)}, \quad (35)$$

where

$$L_n = -\frac{d_{1n}^2}{2\beta} \left( e^{-2ik_n \beta h} - 1 \right) + \frac{d_{2n}^2}{2\beta} \left( 1 - e^{2ik_n \beta h} \right) - 2ik_n h d_{1n} d_{2n}, \quad (36)$$

$$\hat{L}_n = -\frac{id_n^2}{2\hat{\beta}}. \quad (37)$$

Similarly, we find the scattered amplitude in the negative  $x_1$ -direction due to the horizontal load

$$A_n^- = \frac{R \left( d_{1n} e^{ik_n \beta x_2^0} + d_{2n} e^{-ik_n \beta x_2^0} \right) e^{ik_n x_1^0}}{2 (C_{55} L_n + \hat{C}_{55} \hat{L}_n)}. \quad (38)$$

#### 4. RESULTS AND DISCUSSION

In this section, numerical calculations are presented to investigate the dispersion characteristics and excitation behavior of Love waves in an orthotropic layered half-space. The considered configuration consists of an orthotropic epoxy layer bonded to an aluminum half-space. Although the formulation is fully general, the numerical examples focus on an orthotropic layer over an isotropic half-space due to the availability of reliable material datasets and the practical relevance of such configurations, e.g., fiber-reinforced polymer layers bonded to approximately isotropic substrates like concrete. Here, the material properties are adopted from Panja et al. (2025), where the orthotropic layer (Epoxy) is characterized by mass density  $\rho = 1600 \text{ kg/m}^3$ , and elastic constants  $C_{44} = 0.337 \times 10^{10} \text{ N/m}^3$ ,  $C_{55} = 0.748 \times 10^{10} \text{ N/m}^3$ , while the isotropic half-space (Aluminum) has mass density  $\hat{\rho} = 2700 \text{ kg/m}^3$ , and material constants  $\hat{C}_{44} = \hat{C}_{55} = 2.65 \times 10^{10} \text{ N/m}^3$ . These parameters are consistent with previously published studies on Love-wave propagation in layered composite media (Panja et al., 2025).

Based on the dispersion relation in Eq. (13), the phase and group velocity curves are computed as functions of the frequency–thickness product, and modal amplitudes are evaluated for a unit-amplitude time-harmonic load. These amplitudes are incorporated into the dispersion diagram to form amplitude-induced scatter plots (Fig. 3).

Fig. 3 shows that the phase velocity of Love waves is bounded between the shear wave velocities of the epoxy layer (lower bound) and the aluminum half-space (upper bound), which is consistent with classical Love-wave theory. The dispersion diagram also reveals multiple guided modes, where only the fundamental mode exists at low frequency–thickness values, and higher-order modes appear as the frequency increases.

The scatter representation highlights that modal contributions are highly non-uniform. Although many modes exist theoretically, only a few exhibit significant amplitudes and dominate

the wave propagation. The excitation efficiency strongly depends on the frequency–thickness parameter, with high-amplitude regions corresponding to effectively excited modes.

Importantly, this amplitude-integrated dispersion diagram provides practical insight for nondestructive evaluation. It enables the direct identification of dominant modes, helping to select optimal frequency ranges, improve signal quality, and reduce unnecessary measurements. Overall, combining dispersion and amplitude analysis offers a more realistic and application-oriented understanding of Love-wave behavior.

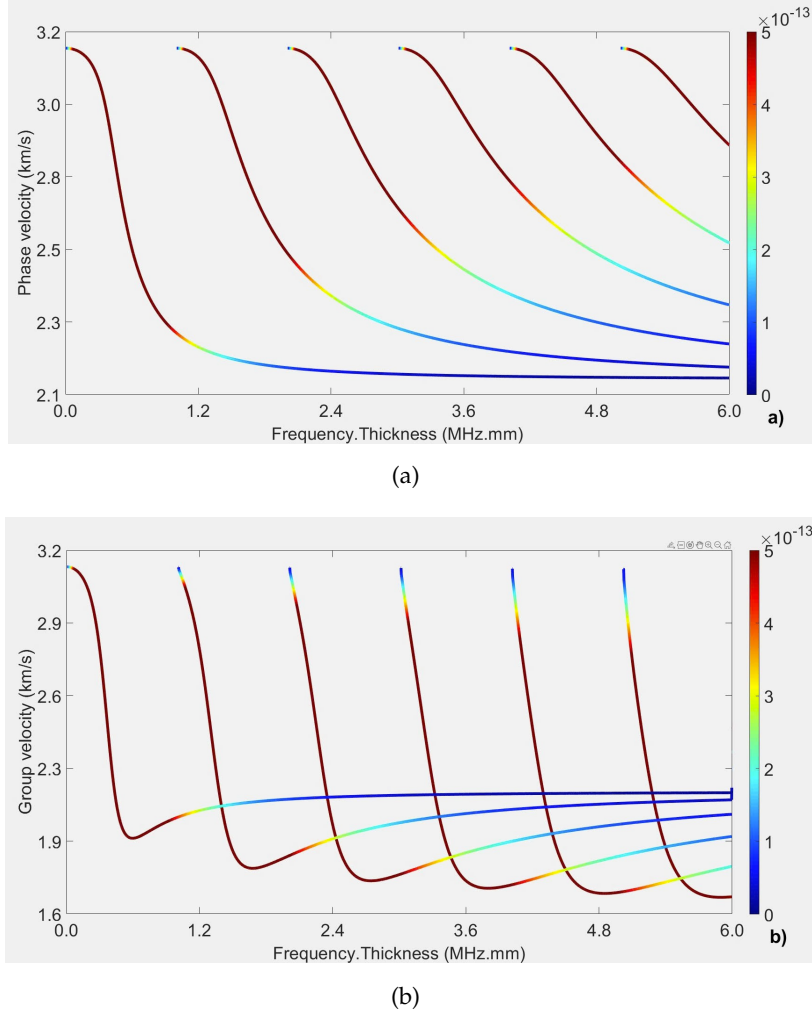


Fig. 3. Phase (a) and group (b) velocity dispersion curves of Love waves

## 5. CONCLUSIONS

This study has developed closed-form solutions for Love-wave propagation and excitation in an orthotropic layered half-space. The free-wave problem was first established within the framework of anti-plane shear motion, and the corresponding dispersion relation was derived from the traction-free and interfacial continuity conditions. The results clearly demonstrate the influence of orthotropic shear stiffness parameters on the dispersion behavior of Love waves. Based on the free-wave solution, the elastodynamic reciprocity theorem was applied to obtain closed-form solutions for the amplitudes of Love-wave modes generated by time-harmonic loading. The proposed approach avoids solving the full boundary-value problem and provides

a physically transparent closed-form formulation incorporating modal orthogonality. The results recover classical isotropic solutions as a limiting case and offer a useful closed-form framework for analyzing guided waves in anisotropic layered media, with potential extensions to multilayered systems and more general loading conditions.

### DECLARATION OF COMPETING INTEREST

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

### CREDIT AUTHOR STATEMENT

Huong Le Thi: *Conceptualization, Methodology, Formal analysis, Writing – original draft.*  
Haidang Phan: *Conceptualization, Validation, Resources, Writing – review & editing, Supervision, Project administration.*

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